

Instructor: B.L. Daku
Time: 15 minutes
Aids: None

Name: _____
Student Number: _____

1. Determine the output of the LTI system defined by

$$h[n] = 2^n u[-n-2],$$

if the input is given by

$$x[n] = 2u[n-2] - 3u[n-9].$$

$$y[n] = \sum_{k=-\infty}^{\infty} 2u[n-2] 2^{n-k} u[-k-n-2] - \sum_{k=-\infty}^{\infty} 3u[n-9] 2^{n-k} u[k-n-2]$$

$$\begin{aligned} y_1: & \quad n \leq 0 \\ & y_1 = \sum_{k=2}^{\infty} 2 \cdot 2^{n-k} \\ & = 2^{n+1} \left(\frac{1}{2} \right)^2 - \left(\frac{1}{2} \right)^0 \\ & = 2^{n+1} \left(\frac{1}{2} \right) = 2^n \end{aligned}$$

$$\begin{aligned} y_2: & \quad n \geq 0 \\ & y_2 = \sum_{k=n+2}^{\infty} 2^{n+1-k} \\ & = 2^{n+1} \left(\frac{1}{2} \right)^{n+2} - \left(\frac{1}{2} \right)^0 \\ & = 2^{n+1} \left(\frac{1}{2} \right)^{n+1} = 1 \end{aligned}$$

$$\begin{aligned} y_2: & \quad n \leq 7 \\ & y_2 = \sum_{k=9}^{\infty} 3 \cdot 2^{n-k} \\ & = 3 \cdot 2^n \left(\frac{1}{2} \right)^9 - \left(\frac{1}{2} \right)^0 \\ & = 3 \cdot 2^n \left(\frac{1}{2} \right)^9 = 3 \cdot 2^{n-8} \\ & = 3 \cdot 2^{n-8} = \frac{3}{2} \\ & \therefore 2^{n-7} = 1 \quad n=7 \text{ critical point} \end{aligned}$$

$$y = y_1 - y_2$$

$$y[n] = \begin{cases} 2^n - 3 \cdot 2^{n-8} & n \leq 0 \\ 1 - 3 \cdot 2^{n-8} & n \leq 7 \\ -1/2 & n \geq 7 \end{cases}$$

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1. Analytically determine the following discrete-time convolution.

$$y[n] = \alpha^n u[n] * \beta^n u[n-2], \quad |\alpha| < 1, |\beta| < 1$$

$$h[n] \quad x[n]$$

$$y[n] = \sum_{k=0}^{\infty} \alpha^k u[n-k] \beta^k u[k-2]$$

$$n \leq 2$$

$$\begin{aligned} n-k > 0 \\ n > k \end{aligned}$$

$$n \geq 2$$

$$\begin{aligned} k-2 > 0 \\ k > 2 \end{aligned}$$

$$n \leq 2$$

$$y[n] = 0$$

$$\begin{aligned} & \sum_{k=0}^{\infty} \alpha^k \beta^{k-2} \\ & = \frac{\alpha^0 + \alpha^1 + \dots}{1 - \alpha^2} \end{aligned}$$

$$\alpha^{-2} \beta^0, \quad \alpha^{-1} \beta^1$$

$$\sum_{k=2}^{\infty} \alpha^k \beta^k$$

$$\begin{aligned} & \sum_{k=2}^{\infty} (\alpha^{-1} \beta)^k \\ & = \frac{(\alpha^{-1} \beta)^2 - (\alpha^{-1} \beta)^{n+1}}{1 - (\alpha^{-1} \beta)} \end{aligned}$$

$$\begin{aligned} & \frac{(\alpha^{-2} \beta^2) - (\alpha^{-n-1} \beta^{n+1})}{1 - (\alpha^{-2} \beta)} \end{aligned}$$

$$y[n] = \frac{\alpha^{-2} \beta^2 - \alpha^{-n-1} \beta^{n+1}}{1 - (\alpha^{-2} \beta)}$$

$$\sum_{k=n}^{\infty} \alpha^k = \frac{\alpha^n}{1 - \alpha}$$

$$\begin{aligned} & \xrightarrow{\quad \text{---} \quad} \\ & \xrightarrow{\quad 0 \quad 1 \quad 2 \quad} \\ & \xrightarrow{\quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad} \\ & \xrightarrow{\quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad} \end{aligned}$$

$$= \frac{\alpha^2 \left(\frac{\beta}{\alpha} \right)^2 - \left(\frac{\beta}{\alpha} \right)^{n+1}}{1 - \left(\frac{\beta}{\alpha} \right)^2}$$